EFFECTS OF MICROMETEOROID CRATERING ON THE DIRECTION OF THE AXIS OF MAXIMUM MOMENT OF INERTIA

H. O. Barthel

REPORT R-274 FEBRUARY, 1966

Coordinated Science Laboratory

UNIVERSITY OF ILLINOIS – URBANA, ILLINOIS

This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy, and U. S. Air Force) under Contract No. DA 28 043 AMC 00073(E).

Portions of this work were also supported by the National Aeronautics and Space Administration under research grant NASA NsG 443.

Introduction

The use of an artifical earth satellite as a gyroscope for measurement of a possible effect of general relativity, described by Schiff,* is being studied. Mirrors on the surface of a fully passive spinning satellite will be used to reflect solar light for measuring a predicted change of six seconds of arc in the spin axis direction during a one year experiment. Although long term changes in the angle between the mirrors and the satellite spin axis can be corrected for, short term changes during the course of a measurement (several weeks) can cause significant errors in the data.

Because of the high degree of inertial symmetry necessary for reduction of gravity-gradient torque, meteoroids which do not significantly disturb the direction of angular momentum have a significant effect, after damping, on the direction of the body axis and mirrors relative to the spin axis. The effect of meteoroids on the principle axes of inertia, which ultimately control the body axis, is discussed herein.

---

*L. I. Schiff, Proc. Natl. Acad. Sci. U.S., 46, 871 (1960)

Effects of Micrometeoroid Cratering on the Direction of the Axis of Maximum Moment of Inertia

H. O. Barthel

This examination of the wandering of the direction of the axis of maximum moment of inertia will proceed in the following way: First, the inertia tensor will be used to determine the effect of the loss of a small mass from the surface. The rotation of axes necessary to yield the principal moments of inertia will be found. Next, the cratering mechanism will be examined briefly and will lead to a consideration of the velocity distribution of micrometeoroids. Having obtained the velocity distribution, an estimate of the fraction of the total flux at a given mass having these velocities will be made. This information will then be used to estimate the number of hits per year, each of which could cause an angular shift of the order of one-tenth of the Schiff precession angle. Finally, a brief discussion of the results will be made.

The inertia tensor for an axis system x,y,z aligned in the direction of the principal moments of inertia of the rotationally symmetric body will be assumed to be

$$\underset{\sim}{I} = \begin{pmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & C \end{pmatrix} \qquad (1)$$

before the loss of the mass m_t at the point x_s, z_s. After the loss of the mass m_t, it becomes

$$\underset{\sim}{I}' = \begin{pmatrix} A - m_t z_s^2 & & -m_t x_s z_s \\ 0 & A - m_t\ (x_s^2 + z_s^2) & 0 \\ -m_t x_s z_s & 0 & C - m_t x_s^2 \end{pmatrix} \tag{2}$$

which can be made principal by the transformation

$$\underset{\sim}{B} = \begin{pmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{pmatrix} \tag{3}$$

where α is found from

$$\tan 2\alpha = \frac{2m_t\ x_s\ z_s}{C\text{-}A\text{-}m_t\ (x_s^2 - z_s^2)}\ . \tag{4}$$

Equation 4, for an oblate spheriod for which $C = \frac{2Ma^2}{5}$, $A = \frac{M}{5}(a^2 + c^2)$ $c = a\sqrt{1 - e^2}$, becomes

$$\tan 2\alpha = \frac{10\ \frac{m_t}{M}\ \frac{x_s}{a} \sqrt{1 - \left(\frac{x_s}{a}\right)^2}\ \sqrt{1 - e^2}}{e^2 - 5\frac{m_t}{M}\ \{2\left(\frac{x_s}{a}\right)^2 - 1 + e^2[1 - \left(\frac{x_s}{a}\right)^2]\}}\ . \tag{5}$$

For small α, $\frac{m_t}{M} \ll e^2$, $\frac{C-A}{C} = \frac{e^2}{2}$, and equation 5 reduces to

$$\alpha = \frac{2.5\ \frac{m_t}{M}\ \frac{x_s}{a} \sqrt{1 - \left(\frac{x_s}{a}\right)^2}\ \sqrt{1 - e^2}}{\frac{C-A}{C}}\ . \tag{6}$$

Equation 6 will be assumed to hold. It can be solved for the mass ratio $\frac{m_t}{M}$ to yield

$$\frac{m_t}{M} = \frac{(\frac{C-A}{C})\ \alpha}{2.5\left(\frac{x_s}{a}\right)\sqrt{1 - \left(\frac{x_s}{a}\right)^2}\ \ \sqrt{1 - e^2}} \tag{7}$$

which has a minimum value when $\frac{x_s}{a} = .707$. Substitution of this value of x_s/a and assuming $\epsilon \ll 1$ results in

$$\frac{m_t}{M}\Big|_{min} \doteq 0.8 \left(\frac{C-A}{C}\right) \alpha \; . \tag{8}$$

The mass m_t lost is dependent upon the hypervelocity impact of a micrometeoroid particle of mass m_p. The micrometeoroid has kinetic energy $\frac{m_p V_p^2}{2}$. This energy is divided between 1) the melting of the mass m_t in a hemispherical crater, 2) the kinetic energy of the mass m_t as it leaves, 3) radiation, 4) the raising of the temperature of the target around the crater, and 5) the generation of waves propagating in the target. In equation form, the melting of m_t can be stated as

$$m_t \, \epsilon = \eta_1 \frac{m_p V_p^2}{2} \tag{9}$$

where ϵ is the energy necessary to melt a unit mass of the target and η_1 is the fraction of the total kinetic energy of the impacting particle going into melting. Values of η_1 are not known. Consequently, values will be assumed when calculations are made.

In order to use equation 9, values of V_p must be known. In much of the literature, it is assumed to be 30 km/sec. However, the distribution of velocities of meteoroids in Fig. 2 of NASA TN D1105 (Davison and Winslow, Space Debris Hazard Evaluation) has led to the following model. Of all particles colliding with the earth, approximately 40% are in retrograde orbits and 60% are in direct orbits. This difference is caused by a decrease in the earth's effective capture cross section as the relative speed increases.

Of those in retrograde orbits, approximately 45% are clustered near a velocity of 38 km/sec relative to the earth and 55% are clustered near 68 km/sec relative to the earth. These groupings arise from the likelihood that, over several billion years, the earth has swept up most of the particles having velocities relative to the sun of 30 km/sec, for these have circular orbits at the earth's distance from the sun. Of those in direct orbits, approximately one-third overtake the earth and have velocities near 15 km/sec relative to the earth and the other two-thirds are overtaken by the earth and have velocities near 22 km/sec relative to the earth.

Letting Φ_i be the flux of particles having masses larger than m_p[calculated from $\Phi_i = 10^{-17}(m_p)^{-1.7}$, where m_p is in grams, Φ_i is in number per square meter per second] and having velocity V_{p_i}, there results from the preceding considerations

$$\Phi = 0.2\Phi_{15} + 0.4\Phi_{22} + 0.18\Phi_{38} + 0.22\Phi_{68} \;. \qquad (10)$$

The number of hits is then simply

$$N = \Phi A T S_f \qquad (11)$$

where A is the surface area, T is the time and S_f is the shielding factor of the earth.

For a solid aluminum sphere (ε = 250 cal/gm) and assuming $\alpha = .1\alpha_{Schiff} = 3.4\times10^{-6}$ rad, η_1 = 1 and 1/4, T = 1 year, S_f = .75 (approximately 750 mile orbit) and $A = 4\pi(.15)^2 = .283\ m^2$, for $\frac{C-A}{C} = 10^{-1}$, and 10^{-2} and 10^{-3} there results

$\frac{C-A}{C}$	$N_{\eta_1 = 1}$	$N_{\eta_1 = 1/4}$
10^{-1}	2.52×10^{-2}	2.38×10^{-3}
10^{-2}	1.25	1.18×10^{-1}
10^{-3}	62.8	5.91

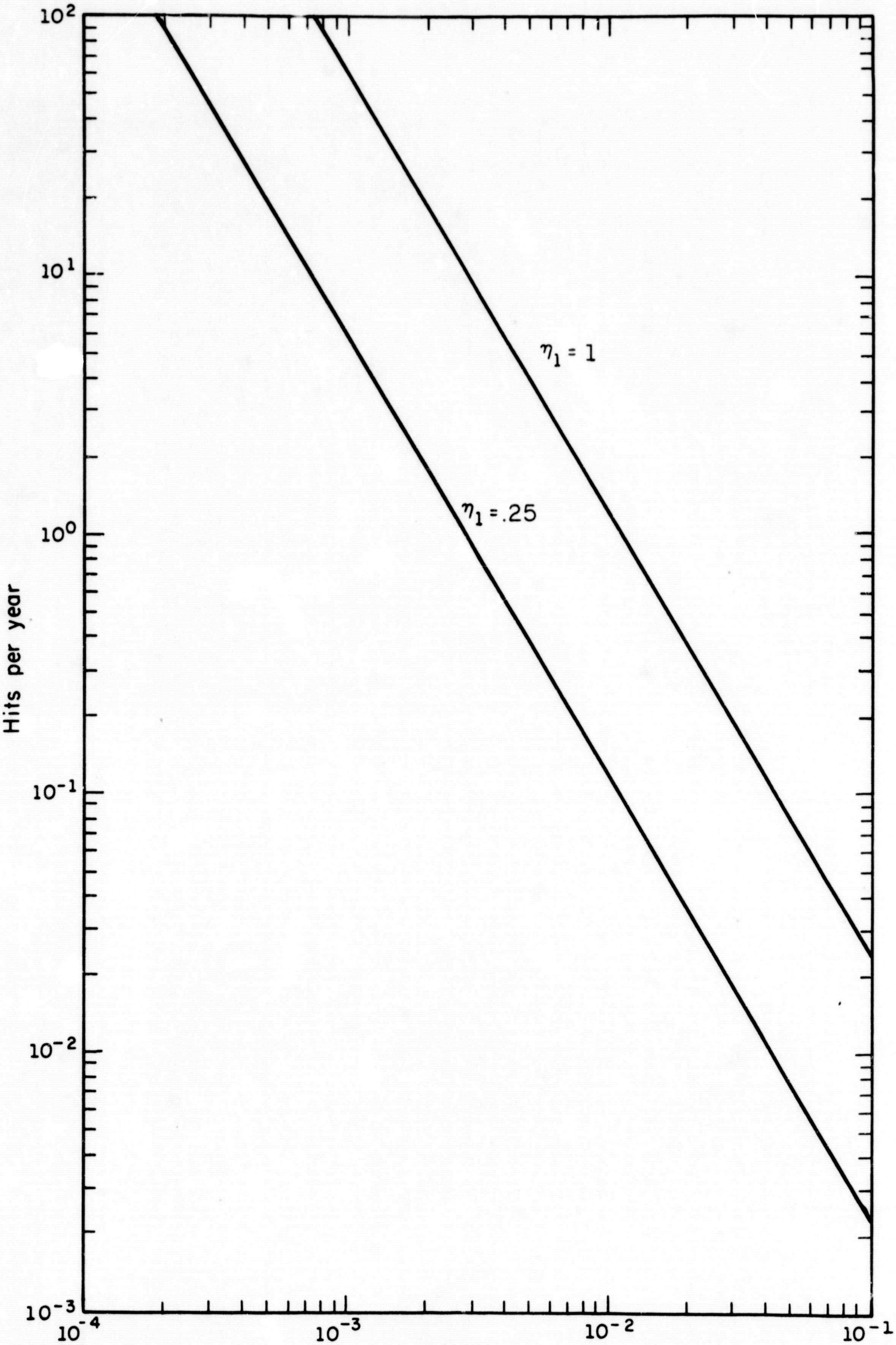
These results are plotted in Fig. 1.

The assumption of $\eta_1 = 1$ should yield an upper bound of hits per year for the assumed average flux levels. On the other hand, the value of $\eta_1 = 1/4$ seemed to be a reasonable estimate in view of the mechanism of cratering. In any case, if a shower of 4 days duration with flux levels 100 times the average flux level occurs, then the yearly total will double.

The main contributions to the yearly total come from the retrograde micrometeoroids. If the distribution of velocities for these small particles is different, or the fraction of the total flux due to these particles is different, then the yearly totals will have to be revised.

In interpreting the results obtained above, one must keep in mind that each hit produces a maximum shift. For purely random hits, 75% of them will produce at least 1/2 of the maximum possible shift. Furthermore, for N greater than 25, there is a 50% chance of having a shift of 5 times the assumed shift. This result arises because in this number of hits one hit will involve a mass which is an order of magnitude larger than the assumed mass.

These considerations lead to the conclusion that the number of hits by masses which cause angular shift of $.1\alpha$ Schiff should not



10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Hits per year
$\frac{C-A}{C}$
$\eta_1 = 1$
$\eta_1 = .25$

Fig. 1

exceed approximately 10 per year. This value also is likely to be reasonably consistent with the optical observation requirements. The detection of these angular shifts and the nutation decay should be accomplished before the next hit. These requirements indicate that $\frac{C-A}{C}$ should be of the order of 5×10^{-3}.

If such a value of $\frac{C-A}{C}$ cannot be used because of other considerations, then with a relatively large number of hits one is led to consider the shifts as a random walk problem. Let each angular shift from a hit be represented as a vector on a plane in which the origin represents the direction of the angular momentum vector and the location of the first point represents the direction of the axis of maximum moment of inertia at insertion. Then, the walk occurs in the following way. The first step will be from the first point toward the origin and its length will be dependent upon the time to the first hit and the nutation decay constant. The second step will be in a random direction and the length will depend upon the probabilities of 1) the striking mass being between m_p and $m_p + dm_p$, 2) the velocity of striking being between V_p and $V_p + dV_p$, and 3) the location and direction (relative to the normal to the surface) of the hit on the body is such that, in terms of the maximum shift for fixed m_p and V_p, the fractional shift will be between F and F + dF. The third step is a decay step toward the origin. The fourth step is similar to the second step. In the continuation of the walk, then all odd steps are toward the origin, all even steps are random. Consequently, one is led to probabilities of being certain distances from the origin after N hits. No attempt has been made to solve this problem.

Security Classification

DOCUMENT CONTROL DATA R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) University of Illinois Coordinated Science Laboratory Urbana, Illinois 61801	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

EFFECTS OF MICROMETEOROID CRATERING ON THE DIRECTION OF THE AXIS OF MAXIMUM MOMENT OF INERTIA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

5. AUTHOR(S) (Last name, first name, initial)

Barthel, Harold O.

6. REPORT DATE	7a TOTAL NO. OF PAGES	7b. NO. OF REFS.
February, 1966	7	---

8a. CONTRACT OR GRANT NO. DA 28 043 AMC 00073(E) b. PROJECT NO. 20014501B31F c. Also National Aeronautics & Space Administration under Grant d. NsG 443	9a. ORIGINATOR'S REPORT NUMBER(S) R-274 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

Qualified requesters may obtain copies of this report from DDC. This report may be released to OTS.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	U. S. Army Electronics Command Fort Monmouth, New Jersey 07703

13. ABSTRACT

The use of an artificial earth satellite as a gyroscope for measurement of a possible effect of general relativity, described by Schiff, is being studied. Mirrors on the surface of a fully passive spinning satellite will be used to reflect solar light for measuring a predicted change of six seconds of arc in the spin axis direction during a one year experiment. Although long term changes in the angle between the mirrors and the satellite spin axis can be corrected for, short term changes during the course of a measurement (several weeks) can cause significant errors in the data.

Because of the high degree of inertial symmetry necessary for reduction of gravity-gradient torque, meteoroids which do not significantly disturb the direction of angular momentum have a significant effect, after damping, on the direction of the body axis and mirrors relative to the spin axis. The effect of meteoroids on the principal axes of inertia, which ultimately control the body axis, is discussed herein.

DD FORM 1473 1 JAN 64

Security Classification

Security Classification

KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
micrometeoroid cratering						
maximum moment of inertia						
inertia tensor						
velocity distribution						
flux						
principal axes of inertia						
body axis						
passive spinning satellite						

INSTRUCTIONS

1. ORIGINATING ACTIVITY: Enter the name and address of the contractor, subcontractor, grantee, Department of Defense activity or other organization (corporate author) issuing the report.

2a. REPORT SECURITY CLASSIFICATION: Enter the overall security classification of the report. Indicate whether "Restricted Data" is included. Marking is to be in accordance with appropriate security regulations.

2b. GROUP: Automatic downgrading is specified in DoD Directive 5200.10 and Armed Forces Industrial Manual. Enter the group number. Also, when applicable, show that optional markings have been used for Group 3 and Group 4 as authorized.

3. REPORT TITLE: Enter the complete report title in all capital letters. Titles in all cases should be unclassified. If a meaningful title cannot be selected without classification, show title classification in all capitals in parenthesis immediately following the title.

4. DESCRIPTIVE NOTES: If appropriate, enter the type of report, e.g., interim, progress, summary, annual, or final. Give the inclusive dates when a specific reporting period is covered.

5. AUTHOR(S): Enter the name(s) of author(s) as shown on or in the report. Enter last name, first name, middle initial. If military, show rank and branch of service. The name of the principal author is an absolute minimum requirement.

6. REPORT DATE: Enter the date of the report as day, month, year; or month, year. If more than one date appears on the report, use date of publication.

7a. TOTAL NUMBER OF PAGES: The total page count should follow normal pagination procedures, i.e., enter the number of pages containing information.

7b. NUMBER OF REFERENCES: Enter the total number of references cited in the report.

8a. CONTRACT OR GRANT NUMBER: If appropriate, enter the applicable number of the contract or grant under which the report was written.

8b, 8c, & 8d. PROJECT NUMBER: Enter the appropriate military department identification, such as project number, subproject number, system numbers, task number, etc.

9a. ORIGINATOR'S REPORT NUMBER(S): Enter the official report number by which the document will be identified and controlled by the originating activity. This number must be unique to this report.

9b. OTHER REPORT NUMBER(S): If the report has been assigned any other report numbers (either by the originator or by the sponsor), also enter this number(s).

10. AVAILABILITY/LIMITATION NOTICES: Enter any limitations on further dissemination of the report, other than those imposed by security classification, using standard statements such as:

(1) "Qualified requesters may obtain copies of this report from DDC."

(2) "Foreign announcement and dissemination of this report by DDC is not authorized."

(3) "U. S. Government agencies may obtain copies of this report directly from DDC. Other qualified DDC users shall request through ______________________."

(4) "U. S. military agencies may obtain copies of this report directly from DDC. Other qualified users shall request through ______________________."

(5) "All distribution of this report is controlled. Qualified DDC users shall request through ______________________."

If the report has been furnished to the Office of Technical Services, Department of Commerce, for sale to the public, indicate this fact and enter the price, if known.

11. SUPPLEMENTARY NOTES: Use for additional explanatory notes.

12. SPONSORING MILITARY ACTIVITY: Enter the name of the departmental project office or laboratory sponsoring (paying for) the research and development. Include address.

13. ABSTRACT: Enter an abstract giving a brief and factual summary of the document indicative of the report, even though it may also appear elsewhere in the body of the technical report. If additional space is required, a continuation sheet shall be attached.

It is highly desirable that the abstract of classified reports be unclassified. Each paragraph of the abstract shall end with an indication of the military security classification of the information in the paragraph, represented as (TS), (S), (C), or (U).

There is no limitation on the length of the abstract. However, the suggested length is from 150 to 225 words.

14. KEY WORDS: Key words are technically meaningful terms or short phrases that characterize a report and may be used as index entries for cataloging the report. Key words must be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location, may be used as key words but will be followed by an indication of technical context. The assignment of links, roles, and weights is optional.

DD FORM 1 JAN 64 1473

Security Classification